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Lepton flavour violation in Little Higgs model with T-Parity

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Abstract

If neutrino mass and mixing consistent with the neutrino oscillation data are the only source of lepton flavor violation (LFV)in nature, the other LFV decays like the radiative and semileptonic decays would be too small to be observed experimentally in the foreseeable future. These decays have been the objects of recent Belle measurments. We analyze LFV in Little Higgs Model with T-parity and find that with reasonable values of the model parameters, these decays can very well be experimentally observable.

1 Introduction

The neutrino oscillation data from the Solar, Atmospheric and Accelerator experiments presents a compelling evidence for the existence of small neutrino mass and large neutrino flavour mixing. The SK atmospheric neutrino and K2K data [1] are best described by dominant $\nu_\mu \to \nu_\tau$ vacuum oscillations with best fit values $|\Delta M_A|^2 = 2.1 \times 10^{-3} eV^2$ and $sin^2 2\theta_A = 1.0$ at 99.73% CL. The Solar neutrino data is described by $\nu_e \to \nu_\mu$ oscillations with best fit value $|\Delta {M_0}^2| = 7.9^{+0.6}_{-0.5} \times 10^{-5} eV^2$ and $tan^2\theta_0 = 0.40^{+.09}_{-.07}$. The Troitzk and Mainz tritium β-decay experiments [2] provide information on the absolute $\bar{\nu_e}$ mass measurment $m_{\bar{\nu_e}} < 2.2$ eV at 95% CL. From the study

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of anisotropy in the CMBR and large scale structure, the WMAP data [3] has severe constraints on the masses of all active neutrino species $\sum m_{\nu_j} < (0.7-1.8)eV$(95% CL).This shows that neutrino flavour is not conserved in nature. In the minimal Standard Model(SM)which has been remarkably successfull in explaining all electro-weak symmetric (EWS)interactions probed so far, the neutrinos are massless because of the restricted particle spectrum and requirement of gauge invariance and renormalizability. One could easily accomodate neutrino mass in the SM by introducing a right handed state resulting in a Dirac mass term through Yukawa coupling just as for charged fermions. This Yukawa coupling of course , has to be roughly six orders of magnitude smaller than for charged fermions in order to obtain small neutrino masses consistent with the above data. This feature is generally considered unnatural. If small neutrino masses in SM are the only source of lepton flavour violation (LFV), other LFV processes like radiative lepton decays ($\mu \to e\gamma$ etc.), semileptonic decays ($\tau \to \mu M$) and trileptonic decays ($\tau \to e^-(\mu^-)\mu^+\mu^-$) which are objects of recent Belle measurments [4], would be so supressed by the small neutrino mass and leptonic GIM mechanism that these observatiions in the foreseeable future are well-nigh impossible. If we simply introduce a neutrino mass in the SM consistent with experiments and a GIM type lepton flavour mixing matrix, the branching ratio of radiative lepton flavour violating decay $\mu \to e\gamma$ has a value less than 10^{-40}. Thus there is a need to explore other sources of LFV in theories beyond the SM.

Neutrino mass generation is not the only problem afflecting SM. The other problem is the so called Hierarchy problem, that is enormous difference between the electro-weak and GUT/Planck scale. The precision electroweak data prefers the existence of light Higgs and thus SM with light Higgs can be considered as an effective theory valid to a high scale perhaps all the way to GUT/Planck scale whereas the Higgs mass is not protected and gets quadratically divergent contribution to its mass and requires fine tuning. Supersymmetry is one of the most attractive framework wherein the quadratic divergences contribution to Higgs mass is cancelled between particles of different statistics at the 1 TeV scale. However with the prospects of LHC drawing near which will test supersymmetry, there have been recently alternative approaches to address this problem. One of the approaches popularly known as Little Higgs Model[5] treats Higgs fields as Nambu-Goldstone bosons of a Global symmetry which is spontaneously broken at some high scale f by acquiring vacuum expectation value (vev). The Higgs field gets a mass through electro-weak symmetry breaking triggered by radiative corrections leading to Coleman-Weinberg type of potential. Since the Higgs is protected by approximate Global symmetry, it remains light and the quadratic divergent contributions to its mass are cancelled between particles of the same statistics. The Littlest-Higgs (LH) model is a minimal model of this genre which accomplishes this task of cancelling quadratic divergence to one loop order with a minimal matter content. The LH model consists of an SU(5) non-linear sigma model which is broken down to SO(5) by a vacuum expectation value f. The gauged subgroup $[SU(2) \times U(1)]^2$ is broken at the same time to diagonal electr-weak SM subgroup $SU(2) \times U(1)$. The new heavy states in this model consist of vector 'top quark' which cancels the quadratic divergence coming from the SM top quark along with the new heavy gauge bosons (W_H, Z_H, A_H) and a triplet Higgs Φ, all of masses of order f and in the TeV range. The effect of these new states on electro-weak precision parameters has been studied to put constraints on the parameters of the model [6]. However, the precision electro-weak observables due to the exchange of heavy degrees of freedom in the model get contributions at the tree level. This requires the cutoff scale of new physics to be $\sim 5 - 10$ TeV and reintroduces the hierarchy problem [6].

Motivated by these constraints, a new implementation[7] of the LH model has been proposed. This is done by invoking a discrete symmetry called the T-parity in the model. T-parity explicitly forbids any tree level contribution from heavy gauge bosons in the e.w. precision observables. It also forbids the interaction that induces the triplet vev. As a result the corrections to e.w. precision observables are generated at the one loop level only. It makes the constraints much weaker than in the tree level case and fine tuning is avoided. In this model SM particles are even under T-parity and most of the new particles at the TeV scale including the Higgs triplet Φ are odd. Another attractive feature of this model is that the lightest T-odd particle (A_H is neutral and if T-parity is conserved, can be a candidate for the dark matter (WIMP) much like the neutralino in MSSM with R-parity.

The particle content of LH model with T-parity consists of [8]

1 T-odd partners of SM gauge bosons W_H, Z_H and A_H with masses $M_{W_H} = M_{Z_H} = gf, M_{A_H} = g\prime f/\sqrt{5}$.

2 T-odd partners of SM fermions (quarks and leptons) with masses typically $\sim \sqrt{2}\kappa f$ of TeV order.

3 A triplet Φ of T-odd Higgs with masses $M_\Phi^2 = \frac{2M_H^2 f^2}{v^2}$

4 A vector T-odd and a singlet T-even top quark with T-odd partner being lighter than the T-even partner both masses being in the TeV range.

5 In addition there is a T-odd doublet $\tilde{\Psi}_R$ and a singlet ξ_R which are needed to cancel two loop quadratic divergence to the Higgs mass but otherwise are assumed to decouple from the spectrum. Their masses are much larger than the symmetry breaking scale ($\sim 5f$) and they have negligible effect on the low energy phenomenology but at the same time their masses are low enough to keep the Higgs mass small.

In this model the T-odd heavy gauge bosons have gauge interactions with the T-odd heavy fermions and T-even SM fermions.The interaction that generates masses of T-odd fermions also couples the T-odd scalar triplet Φ to SM fermions through $\tilde{\Psi}_R$ through the Yukawa coupling. These interactions can be extended to include generation mixing through CKM type matrices just as in SM .The generations would now mix and unless we have a universally degenerate mass spectrum for the T-odd fermions, the interaction would result in FCNC and LFV [9].

In the notation of Hubisz et.al[8], the interactions are given by

$$\mathcal{L}_G = g\bar{\Psi}_{Hi}V^{\dagger i}_{Hj}G_H V^j_{SMk}\Psi^k_{SM} + hc \tag{1}$$

$$\mathcal{L}_Y = \kappa^i_j f(\bar{\Psi}_{2i}\xi\tilde{\Psi}^j + \bar{\Psi}_{1i}\Sigma_0\Omega\xi^\dagger\Omega\tilde{\Psi}^j) + hc \tag{2}$$

where the rotation matrices are related to the appropriate CKM matrix as in SM and Ψ_{SM} and Ψ_H are T even and odd fermion doublets.

2 Lepton Flavour Violation

Radiative leptonic decays $\mu \to e\gamma, \tau \to e\gamma$ along with semileptonic decays $\tau \to \mu\pi(\eta, \eta\prime, K)$ which are objects of recent Belle measurments [4] are sensitive probes of LFV. In order to obtain information on LFV couplings we have to confront anomalous magnetic moment of the muon which is by far the most precisely measured quantity in nature.

a) Anamolous magnetic moment of muons

Theoretical predictions of SM for $a_\mu = \frac{g-2}{2}$ with experimental results [10] give

$$a_\mu(E821) - a_\mu(SM) = (25.2 - 26.0 \pm 9.4) \times 10^{-10} \tag{3}$$

In LH model with T-parity the contributions to a_μ comes from the exchange of heavy vector bosons and Higgs triplet Φ. We have calculated the contribution from the exchange of these particles in the Unitary gauge and obtain

$$a_\mu(W_H) = -\frac{g^2}{32\pi^2}\frac{m_\mu^2}{M_{W_H}^2}\Sigma V_{H2i}^* V_{Hi2} F_{W_H}\left[Z_i = \left(\frac{M_{\nu_{Hi}}}{M_{W_H}}\right)^2\right] \tag{4}$$

$$a_\mu(Z_H) = \frac{g^2}{32\pi^2}\frac{m_\mu^2}{M_{Z_H}^2}\Sigma V_{H2i}^* V_{Hi2} F_{Z_H}\left[Z_i = \left(\frac{M_{l_{Hi}}}{M_{W_H}}\right)^2\right] \tag{5}$$

$$a_\mu(A_H) = a_\mu(Z_H)[g \to g\prime/5, M_{Z_H} \to M_{A_H}] \tag{6}$$

where

$$F_{W_H}(Z_i) = \frac{1}{6(Z_i-1)^4}[-10+37Z_i-48Z_i^2+7Z_i^3+14Z_i^4+(-12Z_i^4-6Z_i^3+24Z_i^2)lnZ_i] \tag{7}$$

$$F_{Z_H}(Z_i) = \frac{1}{12(Z_i-1)^4}[-8+38Z_i-39Z_i^2+14Z_i^3-5Z_i^4+18Z_i^2)lnZ_i] \tag{8}$$

Contribution of Φ involves exchange of mirror leptons ($\tilde{L}$) which are supposed to decouple

$$a_\mu(\Phi^-) = \frac{\kappa^2}{32\pi^2}\frac{m_\mu^2}{M_\Phi^2}\Sigma V_{H2i}^* V_{Hi2} F_\Phi\left[Z_i = \left(\frac{M_{\tilde{\nu}_{Hi}}}{M_\Phi}\right)^2\right] \tag{9}$$

$$F_\Phi(Z_i) = \frac{1}{6(Z_i-1)^4}[-1-3Z_i^2-2Z_i^3+6Z_i^4+6Z_i^2)lnZ_i] \tag{10}$$

From eqns. (4) and (5) in the limit of small neutrino mass , we get the SM contribution to a_μ given by

$$a_\mu(SM) = \frac{g^2}{48\pi^2}\frac{m_\mu^2}{M_{W_L}^2}\{\frac{5}{2}-(1+2S_W^2-4S_W^4)\} \tag{11}$$

For representative values of masses of T-odd particles discussed above, we find that the contribution of T-odd vector bosons, fermions and scalars is not more than a few percent of the SM contribution and therefore well within control.

b) Radiative decays

As discussed above if small neutrino mass and large mixing as required by neutrino oscillation data are accomodated in SM by simply introducing small neutrino mass, other LFV process $\mu \to e\gamma$ will have a branching ratio $< 10^{-40}$ so small that there would be no hope of detecting this in the foreseeble future. In LH model with T-parity there is a possibility of branching ratio being enhanced even with a TeV level GIM mechanism. Present experimental limits on radiative decays of leptons at 90 % CL [4] are

$$Br(\mu \to e\gamma) < 1.2\times 10^{-11}, Br(\tau \to \mu\gamma) < 6.8\times 10^{-8}, Br(\tau \to e\gamma) < 3.92\times 10^{-7} \tag{12}$$

Branching ratios can be easily calculated and we get

$$BR(W_H) = \frac{3}{2}\frac{\alpha}{\pi}\left(\frac{M_{W_L}}{M_{W_H}}\right)^4 \delta_{W_H}^2 = 8.15\times 10^{-7}\delta_{W_H}^2 \tag{13}$$

$$BR(Z_H) = \frac{3}{8}\frac{\alpha}{\pi}\left(\frac{M_{W_L}}{M_{Z_H}}\right)^4 \delta_{Z_H}^2 = 2.04\times 10^{-7}\delta_{Z_H}^2 \tag{14}$$

$$BR(A_H) = \frac{3}{200}\left(\frac{g'}{g}\right)^4\frac{\alpha}{\pi}\left(\frac{M_{W_L}}{M_{A_H}}\right)^4 \delta_{A_H}^2 = 2.57\times 10^{-7}\delta_{A_H}^2 \tag{15}$$

$$BR(\Phi) = \frac{3\alpha}{\pi}\left(\frac{\kappa}{g}\right)^4\frac{M_{W_L}^4}{\tilde{M}_{\nu_H}^2 m_\mu^2}\delta_\Phi^2 = 8.15\times 10^{-7}\delta_\Phi^2 \tag{16}$$

In SM we have

$$BR(SM) = \frac{3\alpha}{32\pi}\delta_\nu^2 < 10^{-40} \tag{17}$$

where

$$\delta_{SM} = \Sigma V_{\mu i}^* V_{ie}\left(\frac{m_{\nu i}}{M_{W_L}}\right)^2 \tag{18}$$

$$\delta_V = \Sigma V_{H\mu i}^* V_{Hie} F_V(Z_V) \tag{19}$$

$$F_{W_H}(Z_i) = \frac{1}{12(Z_i-1)^4}[10 - 43Z_i + 78Z_i^2 - 49Z_i^3 + 4Z_i^4 + 18Z_i^3 lnZ_i] \tag{20}$$

$$F_{Z_H}(Z_i) = \frac{1}{12(Z_i-1)^4}[-8 + 38Z_i - 39Z_i^2 + 14Z_i^3 - 5Z_i^4 + 18Z_i^2) lnZ_i] \tag{21}$$

$$F_\Phi(Z_i) = \frac{1}{6(Z_i-1)^3}[-1 + Z_i^2 - 2Z_i lnZ_i] \tag{22}$$

Substituting these in the branching ratio expressions, we find that if order one mixing angles are allowed in the heavy fermion sector , a Tev scale GIM suppression is necessary and even a mass spectrum which is degenerate upto few percent is enough to make the branching ratios accessible to the present experimental limits [4]

c) Semi-leptonic decays

Recent experimental searches by Belle [4] give

$$BR(\tau \rightarrow \mu\pi) < 4.1\times 10^{-7}, BR(\tau \rightarrow \mu\eta) < 1.5\times 10^{-7}, BR(\tau \rightarrow \mu\eta\prime) < 4.7\times 10^{-7} \tag{23}$$

In the LH model with T-parity, we get new contributions to the semileptonic LFV decays. These contributions come from Box diagrams that contain T-odd heavy gauge bosons, heavy quarks and leptons. The LFV arises mainly because of generation mixing in the heavy leptonic sector. The dominant contribution to the LFV effective Hamiltonian can be estimated following $\Delta S = 2$ effective Hamiltonian calculations for $K^0 - \bar{K}^0$ mixing [11]. The Hamiltonian has V-A structure and the leading term is proportional to $\frac{v^2}{f^2}$ and can be written as

$$\mathcal{H}_{eff} = \frac{G_F^2}{64\pi^2} M_{W_L}^2 \frac{v^2}{f^2} \sum \lambda_i \lambda_j A_{ij}(Z_i, Z_j)(\bar{d}d)_{V-A}(\bar{\mu}\tau)_{V-A} \tag{24}$$

which can be written as

$$\mathcal{H}_{eff} = \frac{1}{\lambda^2}(\bar{d}d)_{V-A}(\bar{\mu}\tau)_{V-A} \tag{25}$$

where

$$\frac{1}{\Lambda^2} \simeq 0.8 \times 10^{-10} \sum \lambda_i \lambda_j A_{ij}(Z_i, Z_j) GeV^{-2} \tag{26}$$

The decay widths can now be easily calculated and we get

$$\Gamma(\tau \rightarrow \mu\pi) \simeq 0.9 \times 10^{-18} |\sum \lambda_i \lambda_j A_{ij}|^2 GeV \tag{27}$$

and

$$BR(\tau \rightarrow \mu\pi) \simeq 0.25 \times 10^{-7} |\sum \lambda_i \lambda_j A_{ij}|^2 \tag{28}$$

Comparing with the experimental data we see that in the LH model with T-parity there is a real possibility of observing these LFV semi-leptonic decays at the present experimental sensitivity for realistic values of generation mixing and even for small departure from the mass degeneracy in the heavy fermion sector.

3 Conclusions

Little Higgs Model with T-parity is an attractive framework which successfully adresses the hierarchy problem and makes the electro-weak precision constraints much weaker. This happens essentially because in this model e.w. precision observables are generated only at the one loop level. The model has the added attraction of providing a possible candidate for dark matter (WIMP)in the form of lightest T-odd neutral gauge boson A_H much like neutralino in MSSM with R-parity. The model can be easily extended to include generation mixing in the heavy fermion sector and can give FCNC and LFV interactions at the observable level. If order one mixing angles are allowed in the heavy fermion sector, a TeV scale GIM supression is necessary and a few percent of departure from mass degeneracy in the heavy lepton mass spectrum is enough to make the lepton flavor violating radiative and semileptonic decays experimentally accessible in the near future.

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